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The American University in Cairo

School of Business

An Application of Copulas in modeling Interest Rate and Equity Returns in the Egyptian Market

A Thesis Submitted to

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In partial fulfillment of the requirements for

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By: Ibrahim Mohamed Abuallail

Under the supervision of:

Dr. Mohamed Bouaddi

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Abstract of The Thesis

An Application of Copulas in modeling Interest Rate and Equity Returns in the Egyptian Market

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The American University in Cairo, 2018

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Despite existing in the statistical literature since the early 20th century, copulas have only started to become widely applied to areas such as finance, economics, and actuarial science in the 21st century. With several economic crises plaguing the world at a greater frequency in the past few decades, copulas have become an invaluable tool in risk management, given their applicability in assessing the co-movement of assets in financial portfolios. However, no attempts to employ these functions in modeling interest rate and stocks in the Egyptian market have been made. This research focuses on assessing the individual as well as the group behavior of a portfolio of bonds and stocks in the Egyptian market across the past decade through finding and fitting the most appropriate marginal distribution functions and a copula function.

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Chapter 1: Introduction

Designing a diversified portfolio to include a combination of interest and equity securities is widely understood by companies and investors looking to decrease risk and increase financial development (Ramcharan 2006). Over the entire past century, the world has witnessed episodes of economic crises, where markets crashed and millions of investors lost a lot of their fortunes. In fact, the period between the 1990s and now witnessed an increase in the frequency of occurrence of such events. The Asian crisis of 1997, the dot-com bubble burst in 2000, the world financial crisis in 2008, were all accompanied in the years in between by many other country or region specific crises. During these years, economic crises occurred in Russia, Argentina, Turkey, Mexico, Uruguay, among many others, all before the Sovereign debt crisis within the EU, in Greece and Portugal. These events led to a growing focus on volatility, with more and more literature shifting focus from answering the question of how to realize the highest possible returns, to the question of how to manage risks and safeguard against volatile movements. But in periods of tranquility and economic growth, a complete focus on avoiding volatility could lead to investment opportunity losses. From this, stems the need to assess the movements of assets and to analyze them on an optimum risk-reward basis, given the investor's level of risk appetite.

The Egyptian market has not at all been different in terms of volatile movements, particularly across the past decade. Asset holders in Egypt have consistently been, and still continue to be, vulnerable to large interest rate and equity movements. The former causes large unrealized gains and losses and can significantly alter solvency positions, just as stock market crashes and booms can. Several major political events that have been going on in Egypt have, at large, affected this volatility. During the past decade, Egypt witnessed uprisings and subsequent leadership changes in 2011 and 2013. Egypt also elected a new parliament, dissolved this parliament, had no parliament for around three years, and then finally elected a new one in 2015. The constitution was changed twice, and the country was hit by several terrorist attacks (albeit mostly confined to the Sinai Peninsula). The Egyptian government implemented an IMF backed reform program, removing subsidies and floating the Egyptian pound. This meant that inflation rates soared, and accordingly the Central Bank raised interest rates to combat this surge in inflation. Finally, worker strikes and nationwide protests were common, adding to a long list of risks investors in Egypt had to consider and endure in their portfolios of assets and securities in the past decade.

In order to assess the risks of investment associated with a portfolio of both interest and equity securities, it is essential to not stop at merely modeling the individual behavior of each variable. Instead, one must also examine the relationship between the movements of both investment instruments, in order to arrive at an optimum risk-return portfolio. In other words, developing a coherent statistical model must not only include a coherent model for the individual risks, but also a coherent dependence function. Modeling this dependence function was in fact the focal point of several studies on several similar volatile emerging markets. In the case of Bogota in Arango et. al (2010), as well as in other cases to be presented, it was found that this specific relationship is seldom linear. This renders the use of simplistic modeling functions and measures such as the Pearson Product correlation in Egypt's case unreliable at best. In fact, the underlying assumption of the correlation is the presence of homoskedasticity, which is almost never present in financial market data due to its nature of volatility clustering and volatility persistence. On the other hand, the general characteristics of time series data, including long memory and asymmetric dependence, make it crucial to develop and employ advanced modeling techniques to be able to more correctly capture the overall volatility.

Finally, it must be noted that there are infinitely many ways in which a set of variables can be dependent. Dependency could be measured not only through the most known measures like the Pearson correlation, Spearman's rho, or Kendall's tau. But also through Blomqvist's coefficient, Gini's coefficient, the measure of Schweizer and Wolf (1981), and more recently the extension of Blest's coefficient by Genest and Plante (2003), among many others. It is one thing to claim all of the above measures is 0, and another to claim independence between the variables. Modern concepts of dependence have become copula-based, allowing for a distinction between the types of association. Some of the most common orderings of dependence are: Positive quadrant dependence (PQD), monotone regression dependence, right-tail increasingness, and left-tail increasingness (Neslehova 2017). For these reasons, this research employs the notion of copulas, which will be discussed at further length in the upcoming chapter, and applies it on Egyptian interest rate yields and EGX30 returns. This provides an inclusive model that captures all forms of interdependency between the variables.

The need for such a model stems from the non-linear nature of relationship that is usually observed between the market for interest rates and equity. This is noted and expanded upon in the upcoming literature review section. In fact, the development of such a model serves

many purposes, including being used in stress test analyses on solvency positions and in forecasting the behavior of one asset class given another.

Chapter 2: Literature Review

This chapter aims to provide an overview of the literature on the behavior of developing economies' financial markets and on several financial market applications of copulas. The first section covers background on the Egyptian market and how the events of the past decade define the risks associated in investments, while the second section focuses on cases where a relationship between both investment choices was calculated for other emerging economies. Finally, the third section covers applications of copulas on several other financial data.

2.1 Egyptian Market in the Past Decade

The Egyptian market has been witnessing several major events across the past decade that make scrutinizing the interlinks between investor behavior and government decisions a very interesting and important study. From revolutions, terrorist attacks, regional instability, to an International Monetary Fund (IMF) reform program leading to major economic restructuring, investments in Egypt seemed to hold a special and a rather fascinating nature.

The year 2011 marked the birth of a large movement for political, social, and economic change in what was labelled the "Egyptian Revolution." The repercussions of this revolution on markets was observed early on, with a suspension of stock market trading for the period of one month (EGX30 Annual Report 2011). The EGX report explained that several precautionary measures were taken by the Egyptian Financial Supervisory Authority (EFSA) in order to regulate the behavior of Egypt's stock market upon resumption of trading, including an enforcement of trading suspensions on stocks moving beyond the 10 percent mark on a daily basis. The report also showed that a shadow of uncertainty was cast on the Egyptian economy during this period of unclear political transition, with stocks plummeting due to large foreign efflux but recovering slightly as investors look forward to the end of this transition period by the end of the year. On the other hand, the Monetary Policy Committee (MPC) decided to hold interest rates constant throughout the year in order to maintain economic growth and bolster the stock market. (Central Bank of Egypt Annual Report 2011). The Central Bank asserted that the MPC finally decided to increase lending, deposit, and discount rates by around 100 basis points towards the end of the year, after realizing the

rebound in the stock market. This highlights a general view of the Egyptian government that interest rates and equity in Egypt seem to be negatively related.

The following years witnessed further major political events that affected the economy significantly. This includes the election of a new president in 2012, only to be ousted a year later for the country to witness another interim period where a new constitution was written and presidential elections were held in 2014. Since then, several terrorist attacks shook the country, negatively affecting tourism and contributing to a foreign currency crisis (World Bank Data). The events also affected foreign direct investments, which plummeted amid security concerns and an overall climate of instability (CAPMAS Data). The foreign currency crisis led to the development of a parallel black market, and to most Egyptian remittances flowing outside of the official channels (Reuters 2017). The economic climate seemed to witness a chain reaction of negativity aggravated by this huge shortage in foreign currency. This reflected in stock market data, albeit with less intensity than the beginning at 2011, as will be seen in the following sections.

All of the aforementioned events led, in one way or another, to a government decision that took an even larger toll on both the stock market and on interest rate movements in Egypt, namely the decision to accept the IMF reform program and to float the Egyptian pound in 2016. Since then, several economic indicators witnessed large changes. Inflation rates soared, the stock market boomed, GDP started to increase, foreign currency reserves increased, all the while the Egyptian pound lost more than half of its value in the matter of a few days (The Economist 2016). Stocks became immediately undervalued overnight, and hence rose significantly. The IMF program also included a gradual removal of subsidies, which further accelerated inflation rates. In the face of a dangerous inflation rate hike, the IMF outlined the necessity of increasing interest rates to absorb liquidity from the market (IMF official Review 2017). The Egyptian Central Bank responded to these recommendations by raising corridor interest rates. On the other hand, all of the aforementioned macroeconomic conditions also significantly affected the behavior of the stock market throughout the course of the following year.

Interest rates peaked towards the middle of 2017, as can be seen in FIGURE 1 below, obtained using Egyptian Central Bank data.

FIGURE 1: Egypt's Central Bank Corridor interest rates

EGYPT INTEREST RATE

SOURCE: TRADINGECONOMICS.COM | CENTRAL BANK OF EGYPT
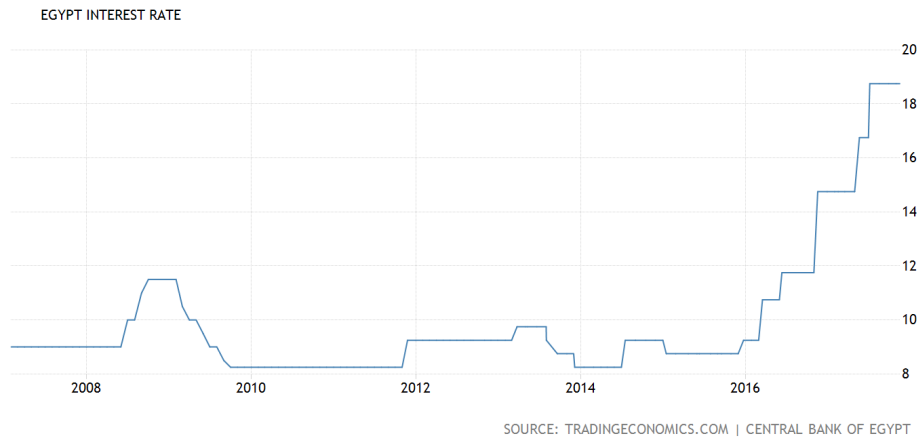
On the other hand, Egypt's main stock index soared following the decision to float, as observed on all trading data websites and as can be seen in FIGURE 2 below:

FIGURE 2: EGX30, Egypt's main stock index, movement across past decade

Egypt EGX 30, M — O 14600.79 H 14726.86 L 14277.47 C 14294.73

This volatility makes investment decisions in the Egyptian market more complex. Building a portfolio to include the optimum amount of interest and equity assets then requires a careful examination of the behaviors of each asset and its co-movement with the rest.

2.2 Relationship Between Interest and Equity in Other Emerging Markets

There is a lot of literature on the relationship between interest rates and equity in several other markets. In fact, there are studies that focus on one aspect or characteristic of one on the other. For instance, Al Nasser and Hajilee (2017) evaluate the impact of interest rate volatility on the development of the stock market in several emerging markets for the period between 1980 and 2011. A dichotomy was drawn between the short run effects and the long run effects, using an ECM model. The findings were rather interesting, with volatility in interest rate being found to negatively affect stock market development in the short run for some markets, and to have the opposite effect in other markets. What was more interesting, was that for some markets, there was a positive relationship between interest rate hikes and stock market development in the short run, but a negative relationship on the long run. These findings point towards a very important story, which is that despite the similarities between the countries studied, the direction of the relationship depended on each country's specific structure.

In fact, the pool of literature on the relationship between interest and equity points in the same direction: the results varied significantly across researches depending on the data used and often depending on the model itself. This is best shown when Khan et. al (2015) used two different models, namely a multiple regression and a VAR(1), to examine the relationship between macroeconomic variables and the stock prices for four emerging South Asian countries. The regression results showed that the direction of the relationship is different between the countries and often different for the same country when assessed using the two different proposed models.

Alam and Uddin (2009) found a negative relationship by looking at a mix of developed and developing countries from 1998 to 2003. Liu and Shrestha (2008) found the same results by comparing interest rates with the index of Chinese stock market. Arango et. al (2010) applied an ESTR-GARCH model and found a non-linear and negative relationship using data from Bogota. Kharwish et. al (2010) also found a negative relationship in Jordan.

On the other hand, Flannery and James (1984) found a positive relationship by using data from the US market. Ologunde et. al (2006) also found a similar relationship in an empirical application on data from Nigeria, where it was found that interest rates indirectly cause positive movements in stock markets through their negative relationship with the government development stock.

2.3 Previous Applications of Copulas

<u>2.3.1 Introduction to Copulas</u>

Copulas were first crystallized as a concept and used by Abe Sklar (1959), where he was able to prove that these functions represent a connection between a joint distribution and their respective marginal distribution functions. Despite existing since then, their popularity and applicability in the fields of actuarial science have only come in the few years after 2000 (Neshlova 2017), which was due to the shift in focus on investment volatility and co-movements of assets explained earlier. According to Sklar's theorem, a copula will take in the Cumulative Distribution Functions (CDFs) of all random variables and include its own parameters, in order to individually and solely capture the dependence between the random variables. On the other hand, the marginal functions solely define the scaling and the shape (mean, skewness, kurtosis, etc). Assuming we have two variables (X, Y), this connection can be seen from the following equation:

$$f_{X,Y}(x, y) = C(F_X(x), F_Y(y); \theta) \cdot f_X(x)f_Y(y)$$

In the case of Gaussian distributions, the copula's parameter indicated with a θ in the above equation becomes the correlation measure ρ. In the case of complete independence, the copula function would return 1, and hence the joint distribution would be the multiplication of the marginal functions.

According to Embrechts (2007), there are many forms of copulas: Archimedean, elliptical, Maltesian, hyperbolic, and zebra copulas. Each of these copula families is characterized by a set of assumptions and properties and must be chosen to model dependencies accordingly.

One of the main benefits of copulas is that regardless of the initial PDFs, if they are continuous functions, a unique copula will exist. This copula will be a joint CDF with uniform margins mapping points on the unit square $(u, v \, E[0,1]x[0,1])$ to values between 0 and 1. Habiboellah (2007) explains that if x and y from any two density distributions were to be transformed into two normally distributed variables u and v, their dependence copula would become a Gaussian copula and can be calculated as follows:

$$C_p(u,v) = \emptyset_p\big(\emptyset^{-1}(u), \emptyset^{-1}(v)\big) = \int\limits_{-\infty}^{\emptyset^{-1}(u)} \int\limits_{-\infty}^{\emptyset^{-1}(v)} \frac{1}{2\pi\sqrt{(1-\rho)}} e^{\frac{-(x^2+y^2-2\rho xy)}{2(1-\rho)}} dxdy$$

Where $\emptyset$ denotes the standard univariate cdf, $\emptyset_p$ denotes the standard bivariate normal cdf, and ρ is the copula's parameter, still taking a value between -1 and +1.

2.3.2 Financial Applications

The literature on copula applications in financial market data is large and growing. One of the main problems researchers face is in fact choosing a specific copula class that meets the nature of financial data. Cherubini et. al (2004) are able to write an entire book describing financial applications of copulas. They give several examples, including using copulas in pricing the forward bivariate digital price of two options. They also include the application of a copulas on the risk of more than one counterparty default, where a Gaussian copula was found to be the best fit for the data. One of the most relevant applications they found, however, was a copula to calculate the risks of investing in two different stock indices (FTSE 100 and DAX 30). They explain that for this stocks dataset, Cherubini and Luciano (2002) had concluded that the best fit for such a copula would be the Frank copula. In fact, Silva Filho et. al (2014) conducted similar work, by assessing the dynamic dependence structure of the US S&P500 stock index, the UK FTSE100, Brazil's BOVESPA, and Mexico's PCMX stock indices. They found the student-t copula to be the best fit for this dataset. Finally, Caillault and Guegan (2005) also conduct a similar analysis in their research, finding that the student-t copula is the best fit copula for a dataset of the "three Tigers" of the Asian market: Thailand, Malaysia, and Indonesia.

Several other authors have done similar work with copulas. Fischer et. al (2009) examine the advantages of different types of copulas in an attempt to arrive at the best one to model dependence structures in the stock market, foreign exchange market, and the commodity markets in Germany. The findings were that pair copulas that were built from bivariate Student t-copulas seemed to have the best fit. Vesper (2012) builds on the idea of static Pair-Copula Construction (PCC) to incorporate multivariate data, and for time dynamic dependence structures. Joe et. al (2012) attempts to incorporate another important aspect in financial data, which is tail asymmetries. Ben Massaoud and Aloui (2015) apply copulas and GARCH copulas on portfolios in an attempt find the optimum risk measure model. The literature, however, contains a huge gap when it comes to modeling the same behavior on Egyptian market data.

Chapter 3: Methodology and Modeling

3.1 Research Problem

As mentioned earlier, there is a major gap in the literature when it comes to the relationship between stock market returns and interest rates specifically in the Egyptian market. In addition, despite consensus on the existence of a significant link between these two variables, there is absolutely no consensus on the nature or direction of this relationship, sometimes even for different papers on same countries (Elton and Gruber 1988). The idea of applying copulas to model this relationship is seldom found in the literature, despite the need for copulas for this in theory. In fact, and as explained earlier, currently the most consistent way in statistics to test for dependence between variables (seeing as dependency can occur in an infinitely large number of ways) is to use a copula.

With very little work done to model stocks and bonds joint behavior in Egypt, this absence of models and information about potential investment assets' co-movement in a very promising market, leaves investment decisions very difficult to conduct. This could mean the lack of realization of full potential, through driving away many would-be investors from the market. Accordingly, this research aims to bridge the gap in the literature by employing the notion of copulas and opening a gateway for information on volatility of Egyptian securities. The main goal is in order to arrive at a more robust and comprehensive model for a portfolio that includes interest and equity assets in Egypt.

3.2 Data

There are two main sources of data used. The first is the Egyptian stock index, EGX30. The second is yields on government bonds with a 6-month maturity found from secondary market auction trading data. This is mainly chosen for two reasons: the short term bonds are more liquid and hence provide enough data points for a more statistically significant modeling process, and 6-month bond yields in Egypt across the past 10 years were far from stable. They included the most volatility among all other maturity classes, meaning that they are far from being risk-free rates. Seeing as the Egyptian bonds are traded on a weekly basis, the frequency of the data is weekly, and hence the returns calculated for both bonds and stocks are to be considered weekly returns. For bonds, the first difference between bond yields was taken in order to calculate the gain/loss of an investor investing in a certain week or waiting to invest in the following week. For stocks, this was done through the logged difference of weekly index prices.

3.3 Modeling

Arriving at a coherent copula based distribution involves a 3-step process whereby the marginal densities for each distribution must be found and specified, then the copula itself must be fitted from among the different copula families and classes. The marginal density of each variable could possibly be fitted parametrically or non-parametrically. The R-package (fitdistrplus) straightforwardly provides an empirical PDF and CDF for each variable. This could provide a starting point to examine what kind of distribution the data could potentially follow. For Bonds, I obtain FIGURE 3 below:

FIGURE 3: The empirical PDF and CDF of bonds data
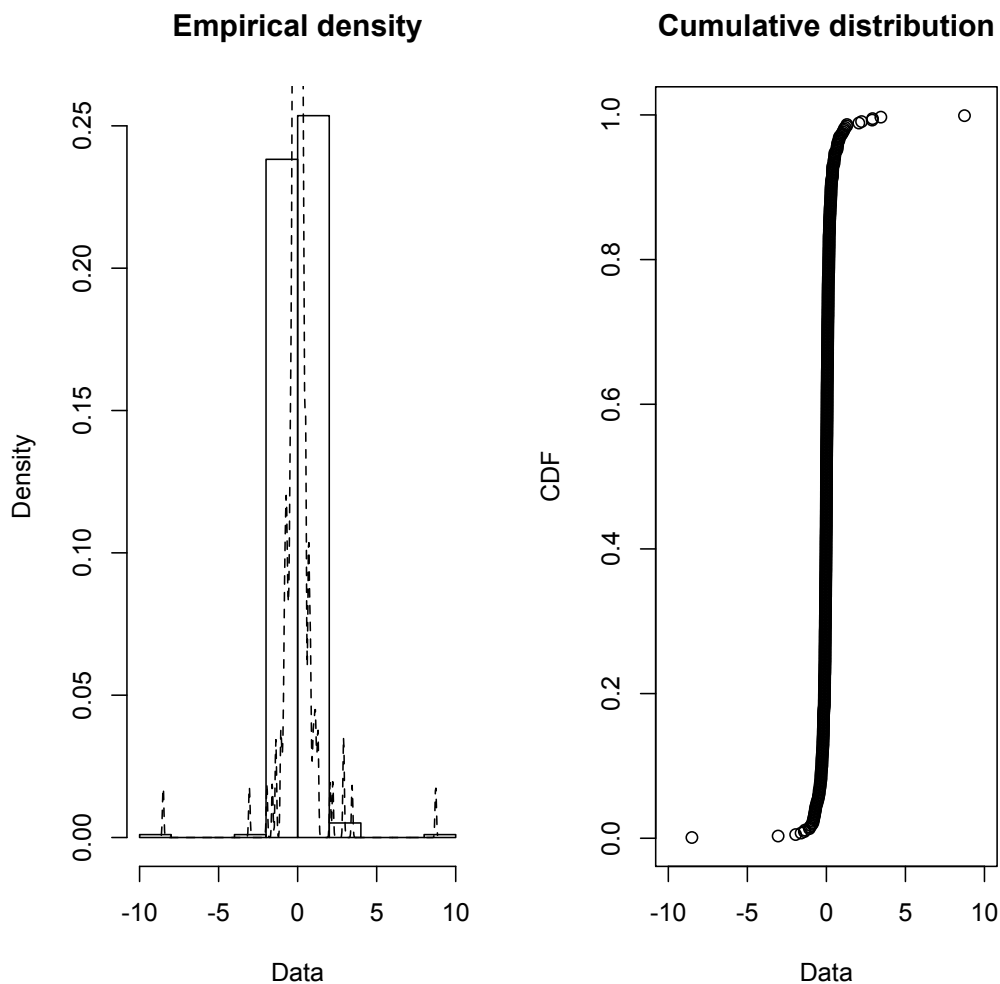
For stocks, I obtain FIGURE 4, below:

FIGURE 4: The empirical PDF and CDF of stocks data

Empirical density   **Cumulative distribution**
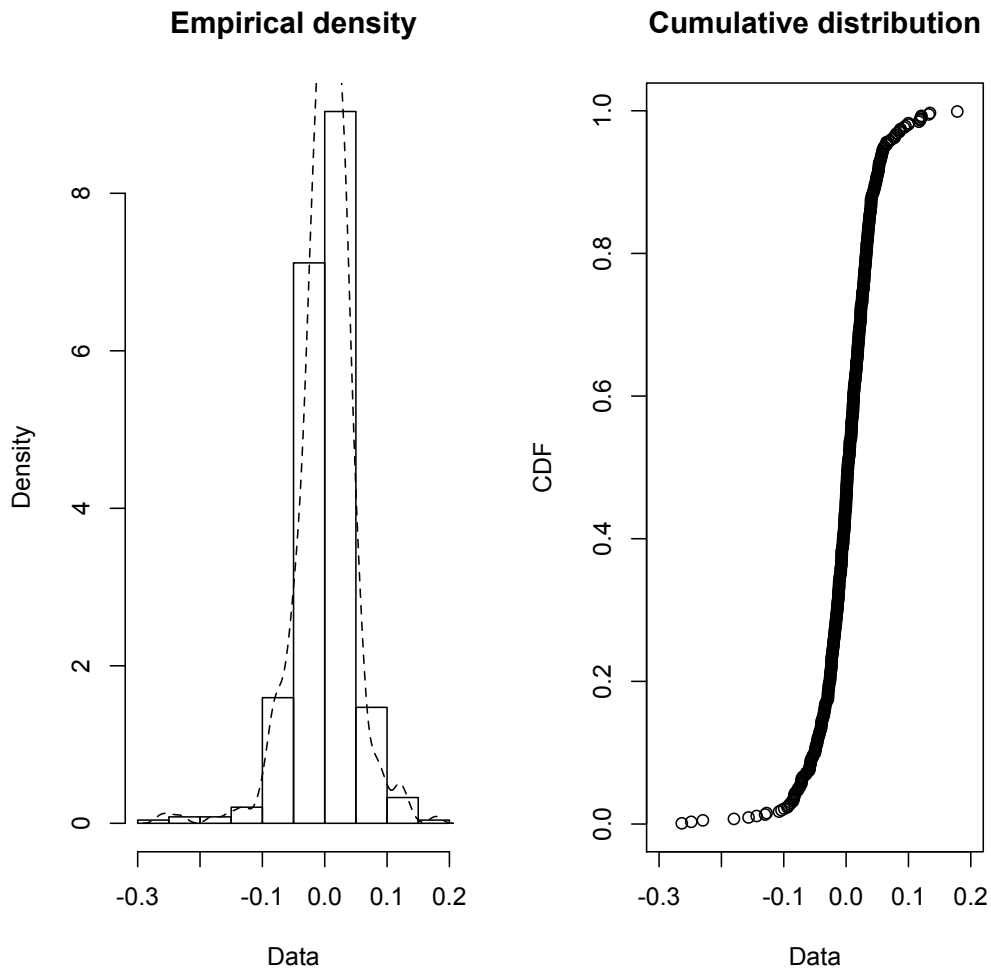
Both of the empirical graphs show that a Gaussian mixture density could be a
potentially good fit for both datasets. However, one must not solely rely on them in deciding
on the most appropriate model. In addition, basic parametric distributions could potentially
offer a more efficient way of capturing and modeling the behavior of each variable. Cullen
and Frey (1999) provide an excellent methodology of assessing different renowned and
commonly used parametric distributions through a bootstrapping technique. The C&F graph
relies on sampled estimates of different skewness and kurtosis values and plots them against
the combinations of those that are theoretically found in different parametric distributions.
The graph can be read by looking at the bootstrapped estimate combination values for
kurtosis and square of skewness, and comparing it to the combinations drawn for several

widely used parametric theoretical distributions. For the stocks data, and as shown in FIGURE 5 below, none of the bootstrapped estimates indicate that the data follows any of the theoretical distributions mentioned. With an overall estimate of around -1 for the square of skewness and 8.5 for the kurtosis, stocks data could not follow the normal, uniform, exponential, logistic, beta, lognormal, weibull, or gamma distributions.

FIGURE 5: C&F Graph with 1000 bootstrapped values for stocks data

Cullen and Frey graph
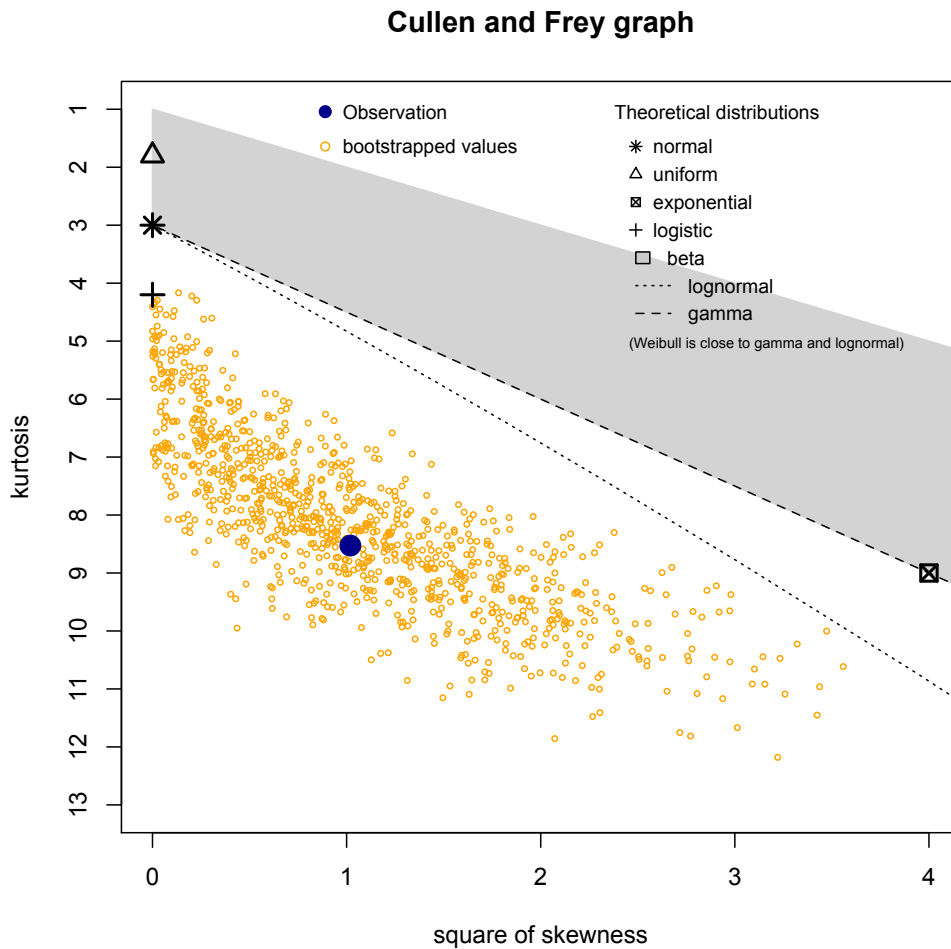
For the bonds data, FIGURE 6 below shows that the majority of bootstrapped estimates of the square of skewness and kurtosis combinations point towards the same conclusion. The overall estimates of around 0.6 for the former and 92 for the latter also indicate that the data could not follow any of the parametric distributions mentioned earlier.

FIGURE 6: C&F Graph with 1000 bootstrapped values for bonds data

Cullen and Frey graph
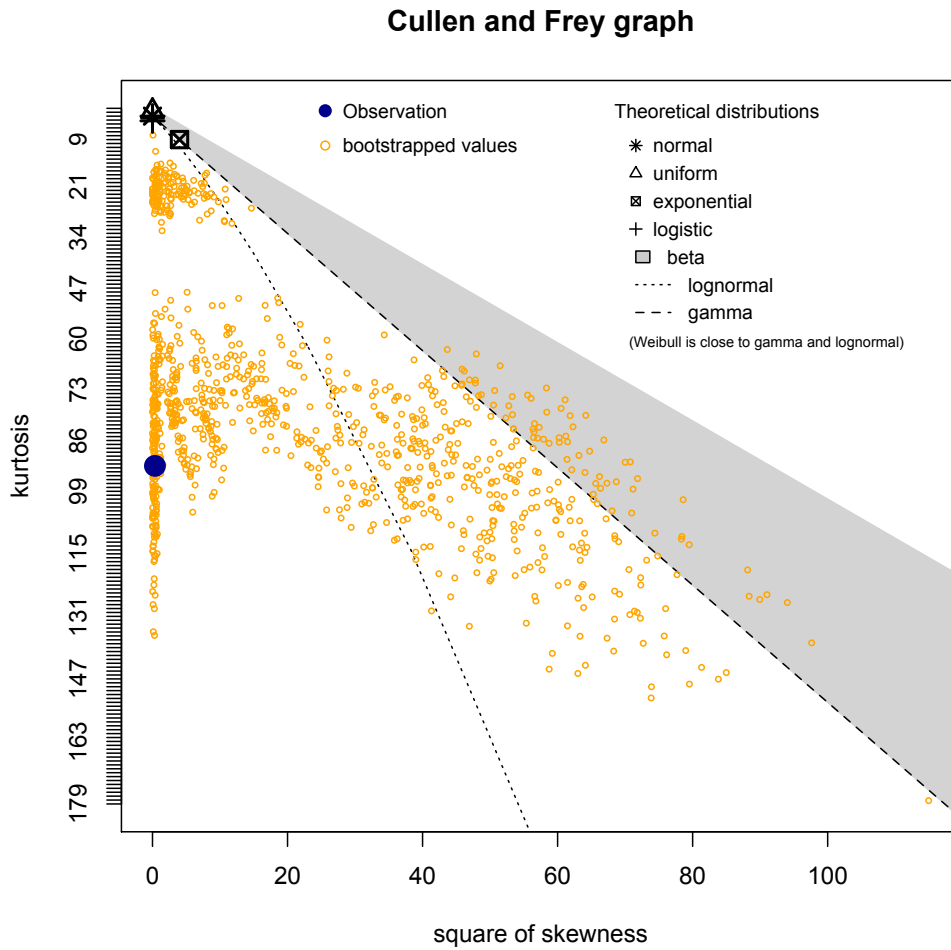
For both datasets, the usage of a Gaussian mixture density then provides a more proper fit for the data. In fact, Carreira-Perpin´ (2000) explain that Gaussian mixtures are universal approximators for continuous densities, meaning that they could be used to approximate any empirical density while still providing the efficiency of parametric distributions. The question then becomes in choosing the most appropriate number of clusters (k). This can be answered by fitting several Gaussian mixtures and assessing a widely used measurement of information criteria, such as the Beysian Information Criterion (BIC). This is calculated as follows:

$$BIC = -2LL + k(\log(n))$$

The LL represents the log-likelihood of the model, while the k represents the number of parameters included in the model. It is clear that within this approach, models with a higher number of parameters are penalized for it through the addition of the k in order to apply the principle of parsimony, and would be required to have a much higher log-likelihood

than those with a lower number of parameters in order to be selected. The model with the lowest BIC score becomes the best fit model for our data. As such, when applied to bonds data with 2, 3, 4, or 5 numbers of clusters, I found:

TABLE 1: Different fitted Gaussian mixture densities for bonds with BIC scores

Number of Clusters	BIC score
2	241.72
3	219.62
4	150.30
5	154.50

This shows that the Gaussian mixture with k=4 (4 clusters) is the best fit for the bonds data. This means that the following is the marginal probability density function for bonds:

$$f(x) = \lambda_1 norm(\mu_1, \sigma_1) + \lambda_2 norm(\mu_2, \sigma_2) + \lambda_3 norm(\mu_3, \sigma_3) + \lambda_4 norm(\mu_4, \sigma_4)$$

where λ represents the weight of each class, and $norm$ represents the PDF of the normal distribution with the given parameters. As for the stocks:

TABLE 2: Different fitted Gaussian mixture densities for stocks with BIC scores

Number of Clusters	BIC score
2	-1695.92
3	-1698.17
4	-1694.37
5	-1692.28

This shows that the Gaussian mixture with k=3 (3 clusters) is the best fit for the stocks data. The PDF for stocks then becomes:

$$f(x) = \lambda_1 norm(\mu_1, \sigma_1) + \lambda_2 norm(\mu_2, \sigma_2) + \lambda_3 norm(\mu_3, \sigma_3)$$

The next step would be to estimate the parameters of these mixture models. This could be done through an Expectation Maximization (EM) algorithm, whereby the

conditional expected log-likelihood is maximized. After finding the estimates of all parameters this way, a parametric bootstrap that produces 1000 bootstrap samples for the parameters is performed in order to calculate the standard errors of the parameters in the mixture distribution. After this is done for bonds, the following was obtained:

Table 3: Bonds parameter estimates with standard errors for each estimate

λ estimate	λ standard error	μ Estimate	μ standard error	σ estimate	σ standard error
0.02581459	0.05525750	0.680204581	0.1123073	3.93879912	0.08920294
0.33072603	0.11882744	0.007661408	0.04241256	0.06840446	0.04680507
0.40362449	0.13376372	0.010921130	0.01400974	0.16943508	0.01809554
0.23983490	0.01048809	-0.014768337	1.211842	0.58505071	1.00190582

As for the stocks, the following was found:

Table 4: Stocks parameter estimates with standard errors for each estimate

λ estimate	λ standard error	μ Estimate	μ standard error	σ estimate	σ standard error
0.03945244	0.14343495	-0.057079770	0.01193366	0.11913369	0.008685192
0.41180437	0.15468073	-0.002688100	0.003045042	0.05247895	0.004262548
0.54874319	0.04306468	0.007338778	0.09448826	0.02416423	0.040194415

The final step in model specifications is to find the most appropriate copula class to fit to the dataset. I start by fitting different widely used copulas and comparing their goodness of fit. The R-package (copula) provides a goodness of fit test that is based on the empirical process of comparing the empirical copula with a parametric estimate of the copula derived under the null hypothesis, as in Genest et. al (2009). Using a parametric bootstrap approach, I am able to estimate the dependence parameters of different copula classes and arriving at a p-value for the fit of the copula. In this case, seeing as the null hypothesis is that the theoretical model does fit the data, a larger p-value would be an indication of a better fitted copula.

Below are the results of fitting several famous elliptical and Archimedean copulas, including the normal, t, gumbel, frank, and clayton:

Table 5: Different fitted copulas with test statistics and P-values

Copula	Test Statistic	P-value
Normal	0.0213	0.4901
Student t	0.0184	0.7376
Gumbel	0.0217	0.5297
Frank	0.0185	0.6782
Clayton	0.0197	0.6593

The results indicate that the bivariate student t is the best copula to use. This is further confirmed by using the R code BiCopSelect() which provides the best fitted bivariate copula to use by comparing the maximum likelihood estimation of each copula. The conclusion is then that the t copula with parameters: rho = -0.04 and degrees of freedom (df) = 7.97, is the best fit for the dataset.

Chapter 4: Conclusion

The world is at a time where investors and researchers are learning the importance of a focus on volatility and on the co-movement of assets, due to an increased frequency and the widespread of economic crises from the late 20th century into the 21st century. With literature racing to cover risk management topics especially in investment, little empirical work has in fact been done on Egypt. The events that have been happening in Egypt across the past decade, both economically and politically, make understanding asset movements and co-movements more crucial now than ever. Failure to do so could result in many business becoming insolvent in the face of huge unrealized losses and volatile equity market. Egypt has been offering very competitive bond returns in order to position itself as one of the most attractive emerging markets to investors. However, little to no work exists to help investors fully understand the risks associated with investing in a portfolio of Egyptian bonds and stocks. Despite the existence of an overall movement towards macroeconomic reforms and legislation meant to improve the overall business environment, these factors of instability and observed volatility, with lack of models available, makes investment decisions in terms of hedging and managing risk an extremely challenging matter.

This research is the first of its kind to be applied on Egyptian stock and interest rate data, and opens a gateway to adjust models on a country specific basis. With more and more comprehensive analysis models on more different kinds of risks that investors face in Egypt, a larger focus could be shifted towards one of the most economically promising investment locations in the world. The research provides a coherent statistical model that captures the two most important elements in portfolio investment: movements and co-movements.

The model found is a multivariate distribution copula based, with marginal distribution functions from a Gaussian mixture with 3 classes for stocks and 4 classes for bonds, with estimates for parameters explicitly shown in table 3 and table 4, and with a t-copula with rho=-0.04 and 7.97 degrees of freedom. This model includes the individual behaviors of each asset class covered by the Gaussian mixture density, as well as a complex copula based component that defines the dependency between them, which was the overall objective of the research.

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Appendix A: R CODES

#Input Data

```
x = read.csv("6m r.csv", header=TRUE, na.string = '.', stringsAsFactors = FALSE);
x=as.data.frame(x)


bonds = x[,1]
stocks = x[,2]
```

#Checking potential distributions to fit:

```
library(fitdistrplus)


plotdist(bonds, histo = TRUE, demp = TRUE)
plotdist(stocks, histo = TRUE, demp = TRUE)


descdist(bonds,boot=1000)
descdist(stocks,boot=1000)
```

#Fitting mixture gaussian distribution:

```
library(mixtools)


bondsmixmdl2 = normalmixEM(bonds,k=2)
bondsmixmdl3 = normalmixEM(bonds,k=3)
bondsmixmdl4 = normalmixEM(bonds,k=4)
bondsmixmdl5 = normalmixEM(bonds,k=5)


stocksmixmdl2 = normalmixEM(stocks,k=2)
stocksmixmdl3 = normalmixEM(stocks,k=3)
stocksmixmdl4 = normalmixEM(stocks,k=4)
stocksmixmdl5 = normalmixEM(stocks,k=5)


(log(nrow(x))*2) - (2 *(bondsmixmdl2$loglik))
```

```r
(log(nrow(x))*3) - (2 *(bondsmixmdl3$loglik))
(log(nrow(x))*4) - (2 *(bondsmixmdl4$loglik))
(log(nrow(x))*5) - (2 *(bondsmixmdl5$loglik))


(log(nrow(x))*2) - (2 *(stocksmixmdl2$loglik))
(log(nrow(x))*3) - (2 *(stocksmixmdl3$loglik))
(log(nrow(x))*4) - (2 *(stocksmixmdl4$loglik))
(log(nrow(x))*5) - (2 *(stocksmixmdl5$loglik))


bondsmixmdl = normalmixEM(bonds,k=4)
stocksmixmdl = normalmixEM(stocks,k=3)


boot.se(bondsmixmdl,B=1000)
boot.se(stocksmixmdl,B=1000)



library(copula)
library(VineCopula)


normcopgf = gofCopula(normalCopula(dim=2),as.matrix(x),N=100,method="SnC")
tcopgf = gofCopula(tCopula(dim=2),as.matrix(x),N=100,method="SnC")
gumbcopgf = gofCopula(gumbelCopula(dim=2),as.matrix(x),N=100,method="SnC")
frankcopgf = gofCopula(frankCopula(dim=2),as.matrix(x),N=100,method="SnC")
claytoncopgf = gofCopula(claytonCopula(dim=2),as.matrix(x),N=100,method="SnC")


var_a = pobs(bonds)


var_b = pobs(stocks)


selectedCopula = BiCopSelect(var_a, var_b, familyset = c(0,1,2,3,4,5,6))


selectedCopula


tcop = tCopula(dim=2, df=7.97, param=-0.04)
```

```r
dbondsgmix <- function(x, p1, mean1, sd1, p2, mean2, sd2, p3, mean3, sd3, p4, mean4, sd4)
{
  p1 * dnorm(x, mean1, sd1) +
  p2 * dnorm(x, mean2, sd2) +
  p3 * dnorm(x, mean3, sd3) +
  p4 * dnorm(x, mean4, sd4)
}


pbondsgmix <- function(x, p1, mean1, sd1, p2, mean2, sd2, p3, mean3, sd3, p4, mean4, sd4)
{
  p1 * pnorm(x, mean1, sd1) +
  p2 * pnorm(x, mean2, sd2) +
  p3 * pnorm(x, mean3, sd3) +
  p4 * pnorm(x, mean4, sd4)
}



qbondsgmix <- function(x, p1, mean1, sd1, p2, mean2, sd2, p3, mean3, sd3, p4, mean4, sd4)
{
  p1 * qnorm(x, mean1, sd1) +
  p2 * qnorm(x, mean2, sd2) +
  p3 * qnorm(x, mean3, sd3) +
  p4 * qnorm(x, mean4, sd4)
}




dstocksgmix <- function(x, p1, mean1, sd1, p2, mean2, sd2, p3, mean3, sd3, p4, mean4, sd4)
{
  p1 * dnorm(x, mean1, sd1) +
  p2 * dnorm(x, mean2, sd2) +
  p3 * dnorm(x, mean3, sd3)
}
```

```r
pstocksgmix <- function(x, p1, mean1, sd1, p2, mean2, sd2, p3, mean3, sd3, p4, mean4, sd4)
{
  p1 * pnorm(x, mean1, sd1) +
  p2 * pnorm(x, mean2, sd2) +
  p3 * pnorm(x, mean3, sd3)
}

qstocksgmix <- function(x, p1, mean1, sd1, p2, mean2, sd2, p3, mean3, sd3, p4, mean4, sd4)
{
  p1 * qnorm(x, mean1, sd1) +
  p2 * qnorm(x, mean2, sd2) +
  p3 * qnorm(x, mean3, sd3)
}

paramMargins <- list(list(p1=0.02581459, mean1=0.680204581, sd1=3.93879912,
                p2=0.33072603, mean2=0.007661408, sd2=0.06840446,
                p3=0.40362449, mean3=0.010921130, sd3=0.16943508,
                p4=0.23983490, mean4=-0.014768337, sd4=0.58505071),
            list(p1=0.03945244, mean1=-0.057079770, sd1=0.11913369,
                p2=0.41180437, mean2=-0.002688100, sd2=0.05247895,
                p3=0.54874319, mean3=0.007338778, sd3=0.02416423))

my_dist <- mvdc(copula = tcop,
          margins = c("bondsgmix", "stocksgmix"),
          paramMargins = paramMargins)

my_dist
```